

# LITERATURE REVIEW OF UNCERTAINTY OF ANALYSIS METHODS

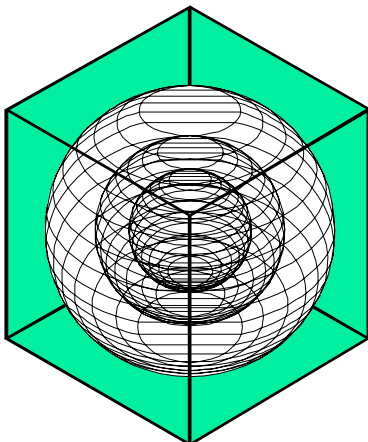
**(Inverse Model Toolkit)**

Report to the  
Texas Commission on Environmental Quality



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## **1 Executive Summary**

This report reviews the reported uncertainty of ASHRAE's Inverse Model Toolkit (IMT) analysis method and the linear, and change-point linear algorithms that it uses by reviewing the published literature on the related accuracy of IMT and its algorithms versus other well-accepted statistical analysis tools, such as SAS. This report begins with a review of the history of the IMT, and the linear and change-point linear models. Then it reviews the published comparisons of the IMT and other analysis software, relying heavily on the accuracy testing that was performed as part of ASHRAE's Research Project 1050-RP. It also includes a detailed description of the basic algorithms and an example of the IMT weather-normalization analysis. In summary, from the literature it was found that the algorithms in the IMT almost exactly reproduce the same regression analysis one would get by running any one of the programs that it was compared against (i.e., usually to several significant digits). Therefore, it can be concluded that the IMT is accurate, when it is called upon to perform weather normalized regressions for modeling building energy use.

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## 2 Introduction

ASHRAE's Guideline 14 has identified three methods for measuring energy savings, including component isolation, before-after measurements, and calibrated simulation (ASHRAE 2002). These methods are intended to be guidelines that will serve as a foundation for the development of reliable and accurate measurement of energy and demand savings from energy conservation retrofits. Guideline 14 describes linear, change-point linear, variable-based degree-day, and multivariable linear regression models as the most used models for calculating before-after savings from energy conservation retrofits.

ASHRAE's Inverse Model Toolkit (IMT) is a FORTRAN 90 application for calculating linear, change-point linear, variable-based degree-day, multi-linear, and combined regression models (Kissock et al. 2002). The development of the IMT was sponsored by ASHRAE research project RP-1050 under the guidance of Technical Committee 4.7, Energy Calculations.

## 3 History of the Inverse Model Toolkit (IMT)

Figure 1 shows the history of the different models contained in the IMT. During the 1980s, Goldberg (1982) and Fels (1986) developed the PRinceton Scorekeeping Method (PRISM) method for use in measuring savings in residential buildings. PRISM uses a Variable-Based Degree Day methods (VBDD) for weather-normalizing the monthly energy use of a residence. The algorithm finds the base-temperature that gives the best statistical fit between energy consumption and the number of variable-base degree-days in each energy use period. Goldberg (1982) developed the mathematical basis of the PRISM model, which includes a detailed uncertainty analysis in her Ph.D. dissertation, "A Geometrical Approach to Non-differentiable Regression Models as Related to Methods for Assessing Residential Energy Conservation," Department of Statistics, Princeton University.

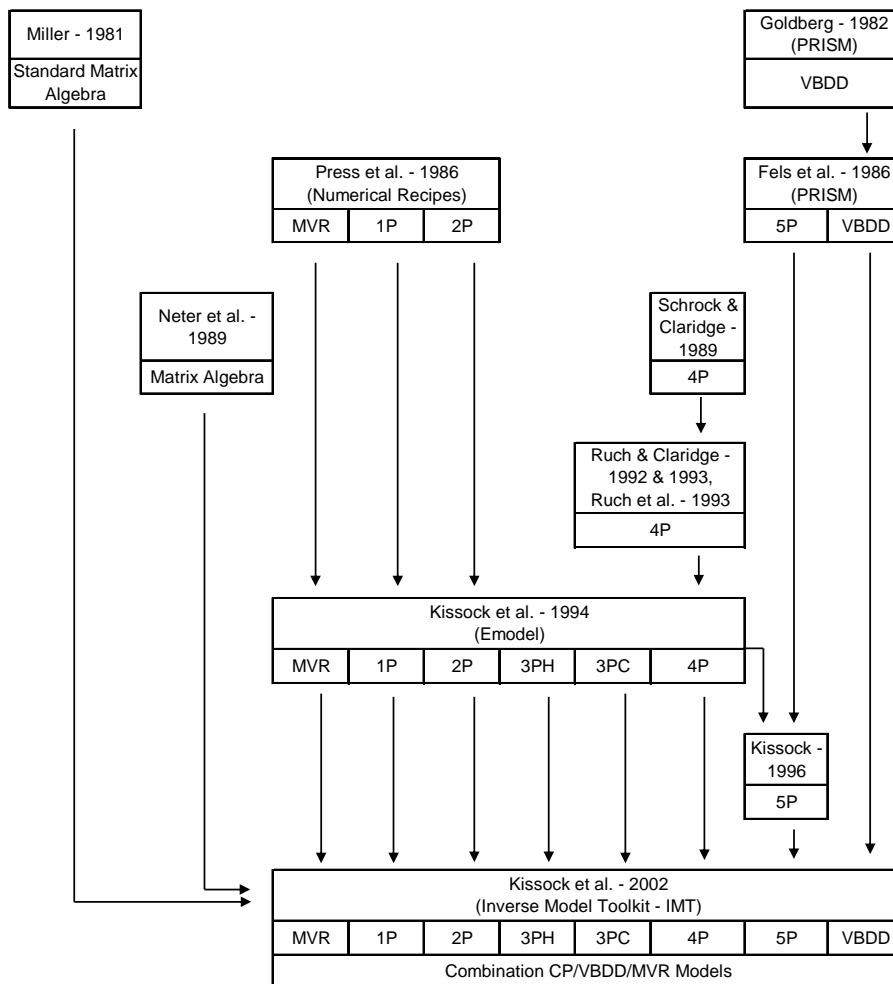
PRISM was one of the first methods to include an estimate of the standard error for all regression parameters (Goldberg, 1982). The method found widespread use in the utility industry, especially in evaluating residential energy conservation programs.

Subsequently, PRISM was found to provide adequate statistical fits with commercial building billing data (Eto, 1988; Haberl and Vajda, 1988; Haberl and Komer, 1990; Kissock and Fels, 1995). However, the physical interpretation of the variable-base degree-day method does not always apply to all commercial buildings that may have varying degrees of heating or cooling energy use (i.e., the energy use is not well described by a three-parameter model), as pointed out by Rabl et al. (1992) and Kissock, (1993).

To resolve this problem, Schrock and Claridge (1989) and later Ruch and Claridge (1992) developed a four-parameter change-point model of energy consumption, along with accompanying error diagnostics for the model's parameters. Their four-parameter change-point model finds the optimal change-point by searching within an interval



known to contain the change-point. Ruch and Claridge (1993) also developed the statistically rigorous methods for estimating Normalized Annual Consumption (NAC) with four-parameter change-point and linear regression models, and investigated how best to incorporate additional variables for the weather normalization using principal component analysis (Ruch et al. 1993).



**Figure 1. History Diagram of the Inverse Model Toolkit.**

Kissock (1993) developed the algorithms for the EModel software as part of his Ph.D. dissertation, which was then developed into the EModel software by Kissock et al. (1994). The algorithms of the software use a two-stage grid search to identify the best change point. In this method, the minimum x value is selected as the initial change point in a standard piece-wise linear regression equation. The change-point is then incremented and the regression is repeated across the range of x-values. The change point that results in the lowest RMSE is selected as the best-fit change-point temperature. This method is then repeated with a finer grid centered about the initial best-fit change point. The uncertainty with which the change-point temperature is known can be approximated as the width of the finest grid. The method is easily adaptable to three-parameter heating,

three-parameter cooling and four-parameter models. The original EModel software also included one-parameter, two-parameter and multi-variable regression models, which used algorithms from Press et al. (1986).

A five-parameter Variable-Based Degree Day (VBDD) model was first reported in Fels (1986) and Fels et al. (1995). An algorithm for five-parameter change-point model was also developed by Kissock et al. (2002). These models have been used extensively with building energy data that have both heating and cooling related loads and have proven to be extremely robust (Haberl et al., 1998).

CP and VBDD models have been shown to provide good statistical fits between building energy use and ambient temperature. However, other variables also influence building energy use. Combination CP-MVR and VBDD-MVR models attempt to retain this ability to describe energy use as a function of ambient temperature while including the effects of additional independent variables. One approach reported in the literature (Rabl and Rialha, 1992; Ruch et al. 1993; Sonderegger, 1997; Sonderegger, 1998) is to sequentially identify the change-point or base temperature and then use this result in a MVR model. An alternative approach is to use indicator variables to produce separate CP or VBDD models for each operating or occupational mode (Austin, 1997; Kissock et al., 1998).

To develop CP-MVR models for Inverse Model Toolkit, the change-point algorithms developed by Kissock (1994, 1996) were extended to include multiple independent variables. Using this approach, CP-MVR models can be identified in a single step, rather than sequentially, and without breaking up the data according to operational modes. The Inverse Model Toolkit can also produce VBDD-MVR models by first running the VBDD model and then running the MVR model on the VBDD residual file.

## **4 Accuracy of the IMT**

### **4.1 Accuracy Testing**

Four sets of accuracy and precision tests (Haberl et al., 2003) were performed as part of the testing for ASHRAE Research Project 1050-RP. The first set of tests was designed to test the accuracy and precision of IMT's computational and regression engines by comparing IMT results with results from the widely used SAS software (SAS 2001). In the second set of tests, IMT 3P, 4P and 5P change-point model results were compared to model results from EModel (Kissock et al., 1994). The primary purpose of this set of tests was to confirm that IMT is finding the same results as the well-tested EModel algorithms. The third set of accuracy tests was designed to see how closely IMT change-point models could identify known change-points and slopes from synthetic data (Sreshthaputra et al., 2001). In the fourth set of accuracy tests, IMT variable-base heating and cooling degree-day models were compared to PRISM HO and CO models (Fels et al., 1995).

## 4.2 Comparisons with SAS

To test the accuracy and determine the precision of IMT's regression algorithms, IMT mean, 2P and MVR models were benchmarked against the statistical software SAS (SAS, 2001). Results from these tests are shown in Table 1, Table 2, and Table 3. The results show good agreement between IMT and SAS to at least three significant figures in the regression parameters tested

## 4.3 Comparison of Change-Point Model Parameters with EModel

The 3P, 4P and 5P change-point model results were compared to model results from EModel (Table 4, Table 5, and Table 6 ). Because IMT and EModel use many of the same algorithms, good agreement between the results was expected. The primary purposes of this test were: 1) to confirm that IMT is finding the same results as the well-tested EModel, and 2) to understand how much the results would vary given that EModel is compiled and executed in Visual Basic and IMT is compiled and executed in FORTRAN.

The data sets used in the mean, 2P, 3P, 4P and 5P comparison tests with SAS were from the Texas LoanSTAR database (Turner et al., 2000). The data sets contain daily energy consumption, temperature, solar and humidity data. The data set used in the MVR comparison with SAS was also from the Texas LoanSTAR database and contains hourly energy consumption, temperature, solar and humidity data from the Texas A&M Zachry Engineering Center. The data sets used in the HDD and CDD comparisons with PRISM are from a residence in College Station, Texas, and contain measured energy consumption and average daily temperature data.

**Table 1. Mean model comparisons between IMT and SAS (Haberl et al., 2003).**

	IMT	SAS	IMT	SAS
Data Set	711	711	963	963
N	356	356	315	315
Mean	25409.281	25409.2809	1118.391	1118.39048
Std dev	2391.109	2391.1088	452.392	452.392131

## 4.4 Comparison of Change-Point Models Parameters with EModel

The 3P, 4P and 5P change-point model results were compared to model results from EModel (Table 4, Table 5, and Table 6 ). Because IMT and EModel use many of the same algorithms, good agreement between the results was expected. The primary purposes of this test were: 1) to confirm that IMT is finding the same results as the well-tested EModel, and 2) to understand how much the results would vary given that EModel is compiled and executed in Visual Basic and IMT is compiled and executed in FORTRAN.

**Table 2. 2P model comparisons between IMT and SAS (Haberl et al., 2003).**

	IMT	SAS	IMT	SAS	IMT	SAS
Data Set	226	226	201	201	952	952
N	364	364	309	309	264	264
R2	0.834	0.8338	0.691	0.6906	0.728	0.728
RMSE	3082.120	3082.12038	5704.015	5704.01365	2065.280	2065.27929
A	-10227.1260	-10229	68439.5078	68439	2338.5520	2338.41956
Std (a)	791.0164	791.03445	1764.5883	1764.58622	595.1532	595.14173
X1	470.2920	470.31234	-649.0869	-649.08587	212.1381	212.13968
Std (X1)	11.0378	11.03807	24.7934	24.79339	8.0110	8.01107
	IMT	SAS	IMT	SAS		
Data Set	207-2	207-2	207-3	207-3		
N	361	361	309	309		
R2	0.861	0.8607	0.691	0.6906		
RMSE	577.486	577.48537	1932.921	1932.92099		
A	-5041.9448	-5041.74522	23192.1055	23192		
Std (a)	154.5213	154.51887	597.9664	597.96592		
X1	105.2378	105.23485	-219.9567	-219.95630		
Std (X1)	2.2346	2.23459	8.4018	8.40175		

**Table 3. MVR model comparison between IMT and SAS (Haberl et al., 2003).**

	IMT	SAS	IMT	SAS	IMT	SAS
Data Set	MVR_1.dat	MVR_1.dat	MVR_2.dat	MVR_2.dat	MVR_3.dat	MVR_3.dat
N	8423	8423	8423	8423	8423	8423
R2	0.438	0.437626	0.873	0.873095	0.582	0.582490
ADJ R2	0.438	0.437425	0.873	0.873019	0.582	0.582242
A	473.3068	473.228982	-2354.9236	-2354.82060	1428.1320	1428.56701
Std (a)	6.8794	6.8800665	35.4836	35.482222	19.8274	19.8343238
X1	4.6356	4.637366	28.4639	28.47099	-12.9370	-12.94613
Std (X1)	0.1385	0.1384824	0.4889	0.488972	0.3020	0.3021457
X2	-1992.5603	-1995.942269	72907.6406	72901.36995	-16776.8887	-16788.49610
Std (X2)	333.7043	333.7266148	1024.9117	1024.973306	723.7756	724.0103164
X3	0.1754	0.175387	-0.0178	-0.01776	0.1297	0.12969
Std (X3)	0.0045	0.0045264	0.0149	0.014853	0.0084	0.0084356
X4			1.8636	1.86297	-0.2883	-0.28844
Std (X4)			0.0327	0.032671	0.0219	0.0219256
X5			0.1787	0.17879	0.0591	0.05928
Std (X5)			0.0188	0.018829	0.0062	0.0062429

**Table 4. 3PC and 3PH model comparisons between IMT and EModel (Haberl et al., 2003).**

	IMT	EModel	IMT	EModel	IMT	EModel	IMT	EModel
Data Set	706	706	208	208	707	707	208-h	208-h
Model	3PC	3PC	3PC	3PC	3PH	3PH	3PH	3PH
N	358	358	358	358	365	365	357	357
R2	0.339	0.34	0.855	0.85	0.934	0.93	0.951	0.95
RMSE	870.641	870.64	4123.559	4123.56	37331.309	37331.31	1821.642	1821.64
Ycp	2417.5938	2417.5941	11145.4775	11145.4541	10248.5869	10248.4245	6001.7227	6001.7111
Std (Ycp)	55.9475	55.9475	**	332.5046	**	2671.7098	**	134.6210
LS	0.0000	0.0000	0.0000	0.0000	-8369.0127	-8369.0158	-639.4753	-639.4761
Std (LS)	0.0000	0.0000	0.0000	0.0000	**	116.5216	7.6691	7.6692
RS	87.6157	87.6158	945.5939	945.5957	0.0000	0.0000	0.0000	0.0000
Std (RS)	6.4898	6.4898	20.6595	20.6596	0.0000	0.0000	0.0000	0.0000
Xcp	56.76	56.76	59.7600	59.7600	61.6800	61.68	79.92	79.92
Std (Xcp)	1.6400	-	1.2600	-	1.6400	-	1.26000	-

**Table 5. 4P model comparisons between IMT and EModel (Haberl et al., 2003).**

	IMT	EModel	IMT	EModel	IMT	EModel
Data Set	706	706	975	975	201	201
Model	4P	4P	4P	4P	4P	4P
N	358	358	279	279	344	344
R2	0.873	0.87	0.816	0.82	0.754	0.75
RMSE	870.641	3861.0432	263.748	263.7481	8051.226	8051.2246
Ycp	17613.2813	17613.7419	1529.8441	1529.8660	27831.1035	27831.8523
Std (Ycp)	**	1678.8280	**	193.4880	**	4807.0528
LS	343.6089	343.6469	16.8140	16.8142	562.6268	562.7605
Std (LS)	28.2882	28.2900	3.0282	3.0284	86.9673	86.9757
RS	1081.8597	1081.8386	73.5243	73.5246	1278.6936	1278.6456
Std (RS)	61.3571	61.3601	5.7500	5.7504	**	154.6463
Xcp	68.5800	68.5800	69.1200	69.12	61.6000	61.6000
Std (Xcp)	1.2600	-	1.1600	-	1.1000	-

**Table 6. 5P model comparisons between IMT and a development version of EModel which includes a 5P model (Haberl et al., 2003).**

	IMT	EModel	IMT	EModel
Data Set	710	710	210	210
Model	5P	5P	5P	5P
N	348	348	362	362
R2	0.274	0.27	0.699	0.70
RMSE	2943.540	2943.5408	11259.003	11259.004
Xcp1	58.7007	58.7007	62.0000	62.0000
std (Xcp1)	3.0340	3.0370	2.3310	2.3333
Xcp2	61.7438	61.7438	69.0000	69.0000
std (Xcp2)	3.034	3.0370	2.3310	2.3333
Ycp	11665.7363	11665.7289	100499.6250	100499.7043
std (Ycp)	**	289.2256	**	966.8825
LS	-120.6786	-120.6790	-635.1901	-635.1859
std (LS)	11.7461	11.7462	**	102.6701
RS	47.0371	47.0376	2534.415	2543.4064
std (RS)	36.6051	36.6051	90.1323	90.1324

#### 4.5 Comparison of Change-point Model Parameters with Known Parameters

In this set of tests, IMT change-point models were derived from synthetic data sets with known slopes and change-points (Table 7, Table 8, Table 9). In all cases, the change-point model parameters were close to the known values, demonstrating IMT's ability to accurately model these data sets. Precise agreement is, of course, not expected because of the search and regression methods used by the toolkit.

These sets of tests also point out important information about how to interpret the standard errors of the IMT change-point and slope coefficients. All IMT change-point algorithms use a two-part grid-search method, in which regression models are identified for successive change-points, until the model (and change point) that produced the lowest RMSE is identified. The standard error of the X change point(s) reported by IMT is one-half of the width of the finest search interval. Thus, it is expected that the true value of the X change point(s) should be within the region defined by the X change point(s) plus or minus the standard error of the X change point(s). The testing reported here confirmed this expectation.

The standard errors of the other coefficients are computed using standard least-square regression methods, and can be interpreted as indicating that, with 68% confidence, the true value of the coefficient is within one standard error of the reported value. However, these methods implicitly assume that the change point(s) is completely known. Thus, the standard errors on the slope and Y change point coefficients reported by IMT reflect the uncertainty of the coefficient due to the scatter of data around the regression line, but underestimate the true uncertainty with which the coefficient is known since the true value of the change-point is not exactly known.

This result is demonstrated in the following tests. Consider, for example, the test results from Table 7 for the 3PC model. IMT reports the right slope as  $-1.0244 \pm 0.0028$ , even though the true slope of  $-1$  is outside of this interval. It is possible that the true slope may be outside of this interval even if the change-point were precisely known. However, in most cases, the reason that the standard errors of the slope and Ycp coefficients underestimate the true uncertainties is because of the uncertainty of the X change point(s).

**Table 7. 3PC and 3PH model comparison between IMT and synthetic data sets with known slopes and change points. Data sets are identified by slope (S) and test (T). Thus, Data Set: S-D, T-12 indicates slope D, test 12 for the indicated model (Haberl et al., 2003).**

	IMT	Known	IMT	Known
Data Set	S-D, T-12	S-D, T-12	S-D, T-12	S-D, T-12
Model	3PC	3PC	3PH	3PH
N	50	50	50	50
R2	1.000		1.000	
RMSE	0.153		0.153	
Ycp	24.9103	25	24.9103	25
std (Ycp)	.0270		.0270	
LS	0.0000	0	-1.0244	0
std (LS)	0.0000		0.0028	
RS	-1.0244	-1	0.0000	-1
std (RS)	0.0028		0.0000	
Xcp	26.7800	25	24.52	25
std (Xcp)	0.9800		0.9800	

**Table 8. 4P model comparison between IMT and synthetic data sets with known slopes and change points. Data sets are identified by slope (S) and test (T). Thus, Data Set: S-B, T-10 indicates slope B, test 10 for the indicated model (Haberl et al., 2003).**

	IMT	Known	IMT	Known
Data Set	S-B, T-10	S-B, T-10	S-F, T-14	S-F, T-14
Model	4P	4P	4P	4P
N	50	50	50	50
R2	1.000		1.000	
RMSE	0.124		0.124	
Ycp	24.7702	25	24.7498	25
std (Ycp)	0.9989		0.0862	
LS	1.0156	1	-0.0156	0
std (LS)	0.0028		0.0028	
RS	0.0133	0	0.9867	1
std (RS)	0.0056		0.0056	
Xcp	24.5200	25	24.5200	25
std (Xcp)	0.9800	-	0.9800	-

**Table 9. 5P model comparison between IMT and synthetic data sets with known slopes and change points. Data sets are identified by slope (S) and test (T). Thus, Data Set: S-D, T-12 indicates slope D, test 12 for the indicated model (Haberl et al., 2003).**

	IMT	Known	IMT	Known	IMT	Known
Data Set	S-D, T-12	S-D, T-12	S-E, T-13	S-E, T-13	S-F, T-14	S-F, T-14
Model	5P	5P	5P	5P	5P	5P
N	50	50	50	50	45	45
R2	0.998		1.000		0.999	
RMSE	0.216		0.147		.184	
Xcp1	15.667	15	15.6667	15	15.6667	16
std (Xcp1)	1.6280		1.6280		1.6280	
Xcp2	30.3333	31	30.3333	30	30.3333	31
std (Xcp2)	1.6280		1.6280		1.6280	
Ycp	15.2201	15	15.1639	15	15.1626	15
std (Ycp)	0.0457		0.0368		0.0390	
LS	0.9558	1	0.9502	1	-1.0169	-1
std (LS)	0.0078		0.0063		0.0066	
RS	-0.9582	-1	1.0159	1	-0.9501	-1
std (RS)	0.0066		0.0053		0.0066	



#### 4.6 Comparisons of HDD and CDD Models with PRISM

The IMT finds the best-fit models between energy use and variable-base heating or cooling degree days. To test the accuracy of the IMT the VBDD models were tested against the PRISM method, the most widely used method of correlating building residential energy use to variable-base degree-days. To accomplish the tests, the IMT's HDD and CDD model results were tested against PRISM HO and CO model results.

Model parameters and coefficients for IMT and PRISM HDD and CDD runs are shown in Table 10. The energy use data are from a residence in College Station, Texas. As can be seen, all parameters and coefficients are in general agreement, considering the different algorithms used by the two methods; however, some comments are called for. First, IMT finds the base temperature to the nearest whole degree, whereas PRISM finds it to at least one decimal place. In addition, PRISM results are reported with a maximum precision of 1 decimal place, thus it is difficult to exactly compare the slopes in the HDD/HO models in Table 10. Finally, PRISM units for base level energy use (A in IMT, alpha in PRISM) are 'energy units per day', whereas IMT units are 'energy units per period'. Thus, to compare IMT and PRISM output, base-level energy use in IMT, A, is divided by the average days per billing period to generate a "corrected" base level energy use A,c. The values for IMT Ac and std(Ac) should, and do, compare to the PRISM's alpha and std(alpha).

**Table 10. Model parameters and coefficients for IMT and PRISM runs (Haberl et al., 2003).**

HDD/HO	IMT	PRISM	IMT	PRISM
Model	HDD	HO	CDD	CO
Data Set	1308ngk	1308ngk	1308elk	1308elk
N	12	12	12	12
R2	0.969	0.977	0.897	0.901
DD Base	73	71.8	70	70.0
X1	0.1368	0.1	1.9555	1.9
Std(X1)	0.0077	0.0	0.2092	0.6
A	18.2518	0.7	476.1488	16.0
Std(A)	2.7116	0.1	53.0032	2.1
A,c	0.5951	0.7	15.6114	16.0
Std(A,c)	0.0884	0.1	1.7378	2.1
days/period	30.67		30.50	

Over twenty other comparisons between IMT and PRISM heating and cooling models were also run, with good agreement between IMT and PRISM on each run. These results are summarized in Table 11 and Table 12 below.

**Table 11. Summary of IMT HDD and PRISM HO comparisons (Kissock et al., 2002).**

Test No.		Data Type	Test	.INS file	Data File	IMT1.6	PRISM
1	VBDD_H0	GAS	HDD	VBDD_H0.ins	VBDD_H0.dat	OK	agree
2	VBDD_H1	GAS	HDD	VBDD_H1.ins	VBDD_H1.dat	OK	agree
3	VBDD_H2	GAS	HDD	VBDD_H2.ins	VBDD_H2.dat	OK	agree
4	VBDD_H3	GAS	HDD	VBDD_H3.ins	VBDD_H3.dat	OK	agree
5	VBDD_H4	GAS	HDD	VBDD_H4.ins	VBDD_H4.dat	OK	agree
6	VBDD_H5	GAS	HDD	VBDD_H5.ins	VBDD_H5.dat	OK	close
7	VBDD_H6	GAS	HDD	VBDD_H6.ins	VBDD_H6.dat	OK	agree

**Table 12. Summary of IMT CDD and PRISM CO comparisons (Kissock et al., 2002).**

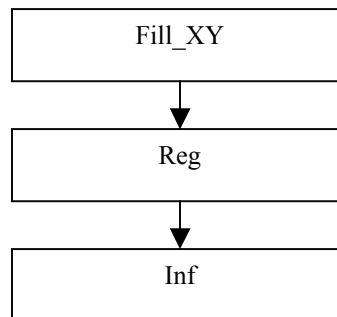
Test No.		Data Type	Test	.INS file	Data File	IMT1.6	PRISM
1	VBDD_C0	WBE	CDD	VBDD_C0.ins	VBDD_C0.dat	OK	agree
2	VBDD_C1	WBE	CDD	VBDD_C1.ins	VBDD_C1.dat	OK	agree
3	VBDD_C2	WBE	CDD	VBDD_C2.ins	VBDD_C2.dat	OK	agree
4	VBDD_C3	WBE	CDD	VBDD_C3.ins	VBDD_C3.dat	OK	agree
5	VBDD_C4	WBE	CDD	VBDD_C4.ins	VBDD_C4.dat	OK	agree
6	VBDD_C5	WBE	CDD	VBDD_C5.ins	VBDD_C5.dat	OK	agree
7	VBDD_C6	WBE	CDD	VBDD_C6.ins	VBDD_C6.dat	OK	agree
8	VBDD_C7	WBE	CDD	VBDD_C7.ins	VBDD_C7.dat	OK	agree
9	VBDD_C8	WBE	CDD	VBDD_C8.ins	VBDD_C8.dat	OK	agree
10	VBDD_C9	WBE	CDD	VBDD_C9.ins	VBDD_C9.dat	OK	agree
11	VBDD_C10	WBE	CDD	VBDD_C10.ins	VBDD_C10.dat	OK	agree
12	VBDD_C11	WBE	CDD	VBDD_C11.ins	VBDD_C11.dat	OK	agree
13	VBDD_C12	WBE	CDD	VBDD_C12.ins	VBDD_C12.dat	OK	agree
14	VBDD_C13	WBE	CDD	VBDD_C13.ins	VBDD_C13.dat	OK	agree

## 5 Numerical Algorithms of IMT

The toolkit is written in FORTRAN 90 and compiled using Numerical Algorithms Group, Inc., NAGWare FTN90 Compiler v2.1x. (Salford, 1996). The executable version of the toolkit, IMT.EXE runs in an MS DOS window of Microsoft Windows operating system. To run IMT.EXE, the dynamic link library files SALFLIBC.DLL and FTN90.DLL must be in the same directory as the IMT.EXE. The source code IMT.F90 is an ASCII text file and can be accessed using any text editor.

### 5.1 Least-Squares Regression Algorithm

All IMT model types except the mean model use least-squares regression to determine the model coefficients. Program flow for the IMT least-squares regression algorithm is shown in Figure 2. Regression begins by calling the Fill\_XY subroutine, which removes records with no-data flags and data that are not in the group specified by the grouping variable, then fills the arrays X and Y. Next, the Reg subroutine calculates least-squares regression coefficients for single or multiple independent variables. The Inf subroutine calculates inference statistics that describe the goodness-of-fit of the model. The Reg and Inf subroutines are described in the following paragraphs.



**Figure 2. Program flow for least squares regression algorithms (Kissock et al., 2002).**

Generalized least-squares regression seeks to estimate model coefficients that minimize the sum of the squared error between predicted and actual observations. The Reg subroutine uses a matrix algebra approach to least squares regression (Neter et al., 1989). In this approach, the matrix of dependent observations,  $Y$ , is equal to the product of the matrix of independent observations,  $X$ , and the matrix of estimated regression coefficients,  $\beta$ , plus an error term,  $E$

$$Y = X\beta + E \quad (1)$$

Solving for  $\beta$  gives:

$$\beta = (X^T X)^{-1} X^T Y \quad (2)$$

The Reg subroutine solves Equation 2 by calling the Trans, Mult and Invert subroutines. The Trans, Mult and Inverse algorithms are simply computational versions of standard matrix algebra (Miller, 1981). In the Trans subroutine, the X matrix is transposed by interchanging the rows and columns. The Mult subroutine performs matrix multiplication in which the elements from one row of the first matrix are multiplied by the elements from the column of the second matrix, then summed. The Invert subroutine finds the inverse matrix of the product of  $X^T X$ .

Before operating on X, however, each X observation is normalized by the mean value of each independent variable. This normalization process provides computational stability for the Invert subroutine in cases where the values of the X observations are very large or very small. Model residuals and inference statistics, such as R<sup>2</sup>, RMSE, CV-RMSE and the standard errors of the regression coefficients, are calculated in the Inf subroutine. To calculate the model residuals, the predicted values of the dependent variable,  $\hat{Y}$ , are computed from:

$$\hat{Y} = X\beta \quad (3)$$

The matrix of residuals, E, is then computed from:

$$E = Y - \hat{Y} \quad (4)$$

The root mean squared error, RMSE, is computed from:

$$RMSE = \sqrt{\frac{\sum (Y - \hat{Y})^2}{(n - p)}} = \sqrt{\frac{Y^T Y - \beta^T X^T Y}{(n - p)}} \quad (5)$$

where n is the number of data observations and p is the number of regression coefficients. The matrix of the standard errors of the regression coefficients, S, is computed from:

$$S = RMSE \sqrt{(X^T X)^{-1}} \quad (6)$$

The squared correlation coefficient, R<sup>2</sup>, is computed from:

$$R^2 = 1 - \frac{\sum (Y - \hat{Y})^2}{\sum (Y - \bar{Y})^2} \quad (7)$$

The adjusted R<sup>2</sup>, is computed from:

$$\text{Adjusted } R^2 = 1 - \frac{(n - 1) \sum (Y - \hat{Y})^2}{(n - p - 1) \sum (Y - \bar{Y})^2} \quad (8)$$

## 5.2 Change-Point Model Algorithm

IMT uses the same algorithm for finding all change-point models, including combination change-point multi-variable regression models (Kissock et al., 1994). Table 13 and Figure 3 contain models, which include steady-state: constant or mean models, models adjusted for the days in the billing period, two parameter models, three parameter models or variable-based degree-day models, four parameter models, five parameter models, and multivariate models.

The steady-state, linear, change-point linear, variable-based degree-day and multivariate inverse models have advantages over other types of models. First, since the models are simple, and their use with a given dataset requires no human intervention, the application of the models can be automated and applied to large numbers of buildings, such as those contained in utility databases. Such a procedure can assist a utility, or an owner of a large number of buildings, identify which buildings have abnormally high energy use. Second, several studies have shown that linear and change-point linear model coefficients have physical significance to operation of heating and cooling equipment that is controlled by a thermostat. Finally, numerous studies have reported the successful use of these models on a variety of different buildings.

### 5.3 One-parameter or constant model.

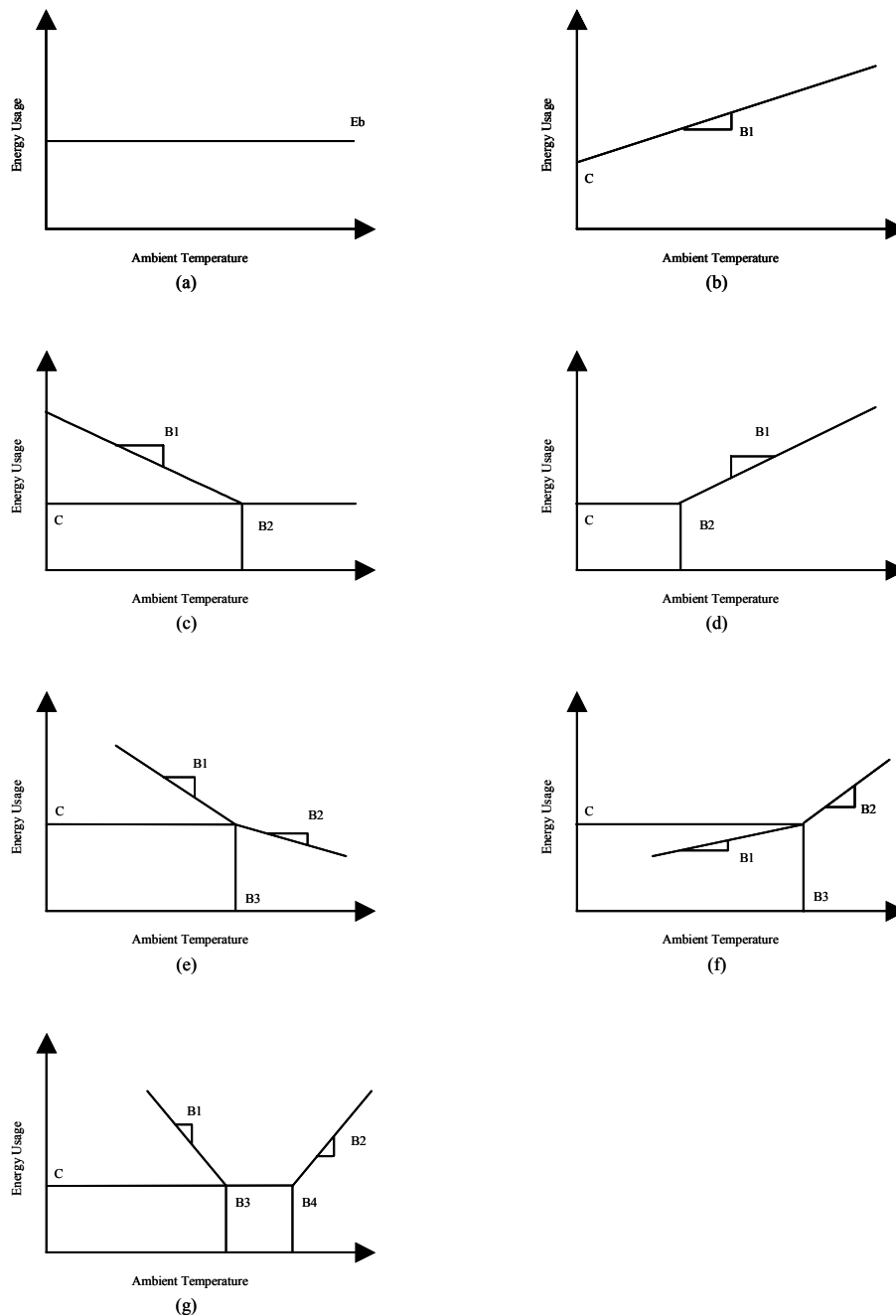
One-parameter, or constant models are models where the energy use is constant over a given period. Such models are appropriate for modeling buildings that consume electricity in a way that is independent of the outside weather conditions. For example, such models are appropriate for modeling electricity use in buildings which are on district heating and cooling systems, since the electricity use can be well represented by a constant weekday-weekend model. Constant models are often used to model sub-metered data on lighting use that is controlled by a predictable schedule.

### 5.4 Day-adjusted model.

Day-adjusted models are similar to one-parameter constant models, with the exception that the final coefficient of the model is expressed as an energy use per day, which is then multiplied by the number of days in the billing period to adjust for variations in the utility billing cycle. Such day-adjusted models are often used with one, two, three, four and five parameter linear or change-point linear monthly utility models, where the energy use per period is divided by the days in the billing period before the linear or change-point linear regression is performed.

**Table 13. Sample Models for the Whole-Building Approach from ASHRAE Guideline 14-2002 (Kissock et al., 2002).**

Name	Independent Variable(s)	Form	Examples
No Adjustment/ Constant Model	None	$E = E_b$	Non weather sensitive demand
Day Adjusted Model	None	$E = E_b \times \frac{\text{day}_b}{\text{day}_c}$	Non weather sensitive use (fuel in summer, electricity in summer)
Two Parameter Model	Temperature	$E = C + B_1(T)$	
Three Parameter Models	Degree days/ Temperature	$E = C + B_1(DD_{BT})$ $E = C + B_1(B_2 - T)^+$ $E = C + B_1(T - B_2)^+$	Seasonal weather sensitive use (fuel in winter, electricity in summer for cooling) Seasonal weather sensitive demand
Four Parameter, Change Point Model	Temperature	$E = C + B_1(B_3 - T)^+ - B_2(T - B_3)^+$ $E = C - B_1(B_3 - T)^+ + B_2(T - B_3)^+$	
Five Parameter Models	Degree days/ Temperature	$E = C - B_1(DD_{TH}) + B_2(DD_{TC})$ $E = C + B_1(B_3 - T)^+ + B_2(T - B_4)^+$	Heating and cooling supplied by same meter.
Multi-Variate Models	Degree days/ Temperature, other independent variables	Combination form	Energy use dependent non-temperature based variables (occupancy, production, etc.).



**Figure 3. Sample Models for the Whole-building Approach. Included in this figure is: (a) mean or 1 parameter model, (b) 2 parameter model, (c) 3 parameter heating model (similar to a variable based degree-day model (VBDD) for heating), (d) 3 parameter cooling model (VBDD for cooling), (e) 4 parameter heating model, (f) 4 parameter cooling model, and (g) 5 parameter model (Kissock et al., 2002).**

### 5.5 Two-parameter model.

Two-parameter models are appropriate for modeling building heating or cooling energy use in extreme climates where a building is exposed to heating or cooling year-around, and the building has an HVAC system with constant controls that operates continuously. Examples include outside air pre-heating systems in arctic conditions, or outside air pre-cooling systems in near-tropical climates. Dual-duct, single-fan, constant-volume systems, without economizers can also be modeled with two-parameter regression models. Constant use, domestic water heating loads can also be modeled with two-parameter models, which are based on the water supply temperature.

### 5.6 Three-parameter model.

Three parameter models, which include change-point linear models or variable-based, degree day models, can be used on a wide range of building types, including residential heating and cooling loads, small commercial buildings, and models that describe the gas used by boiler thermal plants that serve one or more buildings. In Table 13, three parameter models have several formats, depending upon whether or not the model is a variable based degree-day model, or three-parameter, change-point linear models for heating or cooling. The variable-based degree day model is defined as:

$$E = C + B_1(DD_{BT}) \quad (9)$$

Where

$C$  = the constant energy use below (or above) the change point,  
 $B_1$  = the coefficient or slope that describes the linear dependency on degree-days,  
 $DD_{BT}$  = the heating or cooling degree-days (or degree hours), which are based on the balance-point temperature.

The three-parameter change-point linear model for heating is described by

$$E = C + B_1(B_2 - T)^+ \quad (10)$$

Where

$C$  = the constant energy use above the change point,  
 $B_1$  = the coefficient or slope that describes the linear dependency on temperature,  
 $B_2$  = the heating change point temperature,  
 $T$  = the ambient temperature for the period corresponding to the energy use,  
 $+$  = positive values only inside the parenthesis.

The three-parameter change-point linear model for cooling is described by

$$E = C + B_1(T - B_2)^+ \quad (11)$$

Where



$C$  = the constant energy use below the change point,  
 $B_1$  = the coefficient or slope that describes the linear dependency on temperature,  
 $B_2$  = the cooling change point temperature,  
 $T$  = the ambient temperature for the period corresponding to the energy use,  
 $+$  = positive values only for the parenthetical expression.

### 5.7 Four-parameter model.

The four-parameter change-point linear heating model is typically applicable to heating usage in buildings with HVAC systems that have variable-air volume, or whose output varies with the ambient temperature. Four-parameter models have also been shown to be useful for modeling the whole-building electricity use of grocery stores that have large refrigeration loads, and significant cooling loads during the cooling season. Two types of four-parameter models are listed in Table 13, including a heating model and a cooling model. The four parameter change-point linear heating model is given by

$$E = C + B_1(B_3 - T)^+ - B_2(T - B_3)^+ \quad (12)$$

Where

$C$  = the energy use at the change point,  
 $B_1$  = the coefficient or slope that describes the linear dependency on temperature below the change point,  
 $B_2$  = the coefficient or slope that describes the linear dependency on temperature above the change point,  
 $B_3$  = the change-point temperature,  
 $T$  = the temperature for the period of interest,  
 $+$  = positive values only for the parenthetical expression.

The four parameter change-point linear cooling model is given by

$$E = C - B_1(B_3 - T)^+ + B_2(T - B_3)^+ \quad (13)$$

Where

$C$  = the energy use at the change point,  
 $B_1$  = the coefficient or slope that describes the linear dependency on temperature below the change point,  
 $B_2$  = the coefficient or slope that describes the linear dependency on temperature above the change point,  
 $B_3$  = the change-point temperature,  
 $T$  = the temperature for the period of interest,  
 $+$  = positive values only for the parenthetical expression.

## 5.8 Five-parameter model.

Five parameter change-point linear models are useful for modeling the whole-building energy use in buildings that contain air conditioning and electric heating. Such models are also useful for modeling the weather-dependent performance of the electricity consumption of variable air volume air-handling units. The basic form for the weather dependency of either case is shown in Figure 3f, where there is an increase in electricity use below the change point associated with heating, an increase in the energy use above the change point associated with cooling, and constant energy use between the heating and cooling change points. Five parameter change-point linear models can be described using variable-based degree day models, or a five parameter model. The equation for describing the energy use with variable-based degree days is

$$E = C - B_1(DD_{TH}) + B_2(DD_{TC}) \quad (14)$$

where

$C$  = the constant energy use between the heating and cooling change points,  
 $B_1$  = the coefficient or slope that describes the linear dependency on heating degree-days,

$B_2$  = the coefficient or slope that describes the linear dependency on cooling degree-days,

$DD_{TH}$  = the heating degree-days (or degree hours), which are based on the balance-point temperature.

$DD_{TC}$  = the cooling degree-days (or degree hours), which are based on the balance-point temperature.

The five parameter change-point linear model that is based on temperature is

$$E = C + B_1(B_3 - T)^+ + B_2(T - B_4)^+ \quad (15)$$

where

$C$  = the energy use between the heating and cooling change points,

$B_1$  = the coefficient or slope that describes the linear dependency on temperature below the heating change point,

$B_2$  = the coefficient or slope that describes the linear dependency on temperature above the cooling change point,

$B_3$  = the heating change-point temperature,

$B_4$  = the cooling change-point temperature,

$T$  = the temperature for the period of interest,

$+$  = positive values only for the parenthetical expression.

## 5.9 Variable-Base Degree-Day Model Algorithm

IMT can find best-fit variable-base degree-day (VBDD) models of the type:

$$Y = \beta_1 + \beta_2 \text{HDD}(\beta_3) \quad (16)$$

$$Y = \beta_1 + \beta_2 \text{CDD}(\beta_3) \quad (17)$$

where  $\beta_1$  is the constant term,  $\beta_2$  is the slope term, and  $\text{HDD}(\beta_3)$  and  $\text{CDD}(\beta_3)$  are the number of heating and cooling degree-days, respectively, in each energy data period calculated with base temperature  $\beta_3$ . The number of heating and cooling degree-days in each energy data period of  $n$  days is:

$$\text{HDD}(\beta_3) = \sum_{i=1}^n (\beta_3 - T_i)^+ \quad (18)$$

$$\text{CDD}(\beta_3) = \sum_{i=1}^n (T_i - \beta_3)^+ \quad (19)$$

where  $T_i$  is the average daily temperature.

To calculate VBDD models, IMT calls two subroutines: FillDNonUni and VBDD. The FillDNonUni subroutine fills and returns the arrays HDD and CDD with the heating and cooling degree days, respectively, for each energy period according to Eqns. 18 and 19.  $\text{HDD}(i,j)$  and  $\text{CDD}(i,j)$  contain the number of degree days in each energy period ( $i$ ) and for base temperatures from 41 to 80 F ( $j$ ).

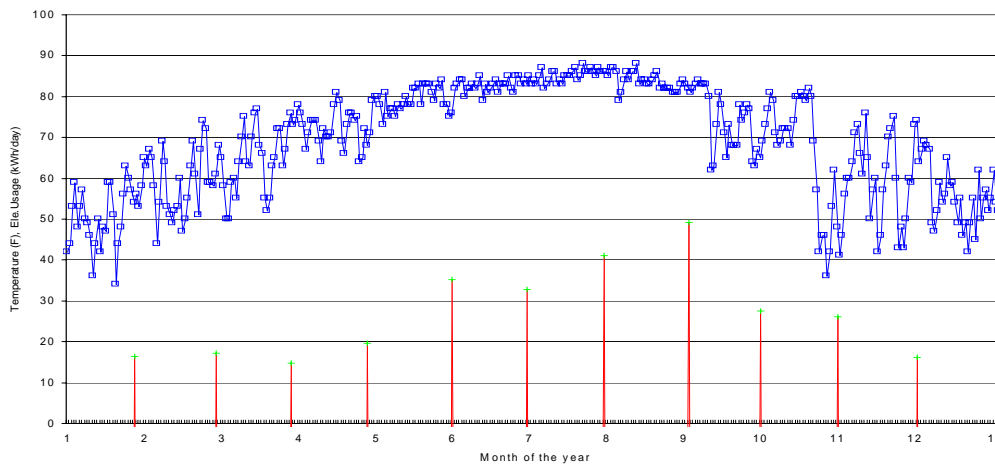
The best-fit VBDD model is identified using a search method by regressing Eqn. 16 or 17 using the HDDs or CDDs in each energy period for successive base temperatures,  $\beta_3$ , from 41 F to 80 F. The base-temperature that results in the model with the highest  $R^2$  is recorded. Eqn. 16 or 17 is regressed once more using the base-temperature that results in the model with the highest  $R^2$ , and the results are reported.

## 6 Example Use of IMT

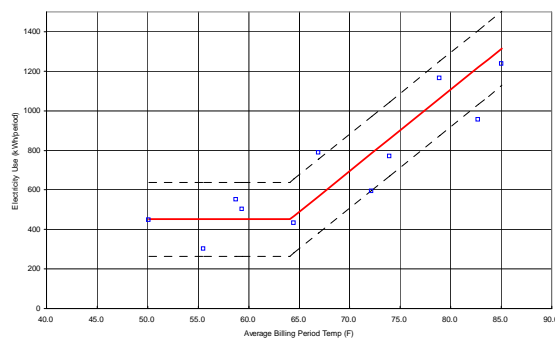
### Example: Whole-building energy use models

Figure 4 presents an example of the typical data requirements for a whole-building analysis, including one year of daily average ambient temperatures and twelve months of utility billing data. In this example of a residence, the daily average ambient temperatures were obtained from the National Weather Service (NWS) (i.e., the average of the published min/max data), and the utility bill readings represent the actual readings from the customer's utility bill. To analyze these data several calculations need to be performed. First, the monthly electricity use (kWh/month) needs to be divided by the days in the billing period to obtain the average daily electricity use for that month (kWh/day). Second, the average daily temperatures need to be calculated from the published NWS min/max data. From these average daily temperatures the average billing period temperature need to be calculated for each monthly utility bill.

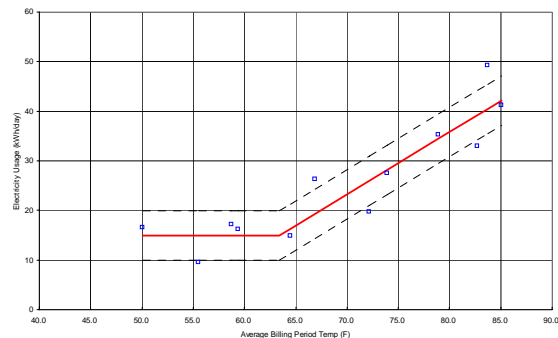
The data set containing average billing period temperatures and average daily electricity use is then analyzed with ASHRAE's Inverse Model Toolkit (IMT) to determine a weather normalized consumption as shown in Figure 5 and Figure 6. In Figure 5 the twelve monthly utility bills (kWh/period) are shown plotted against the average billing period temperature along with a three-parameter change-point model calculated with the IMT. In Figure 6 the twelve monthly utility bills, which were adjusted for days in the billing period (i.e., kWh/day) are shown plotted against the average billing period temperature along with a three-parameter change-point model calculated with the IMT. In the analysis for this house, the use of an average daily model improved the accuracy of the unadjusted model (i.e., Figure 5) from an  $R^2$  of 0.78 and CV(RMSE) of 24.0% to an  $R^2$  of 0.83 and a CV(RMSE) of 19.5% for the adjusted model (i.e., Figure 6), which indicates a significant improvement in the model.



**Figure 4: Example Data for Monthly Whole-building Analysis (upper trace, daily average temperature, F, lower points, monthly electricity use, kWh/day) (Haberl and Culp, 2004).**

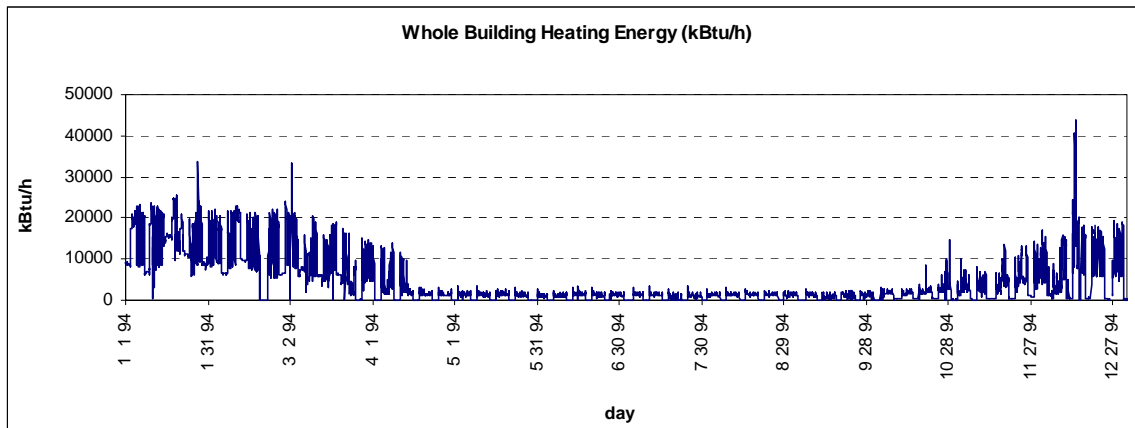


**Figure 5. Example Unadjusted Monthly Whole-building Analysis (3P Model) for kWh/period ( $R^2 = 0.78$ , CV(RMSE) = 24.0%).**

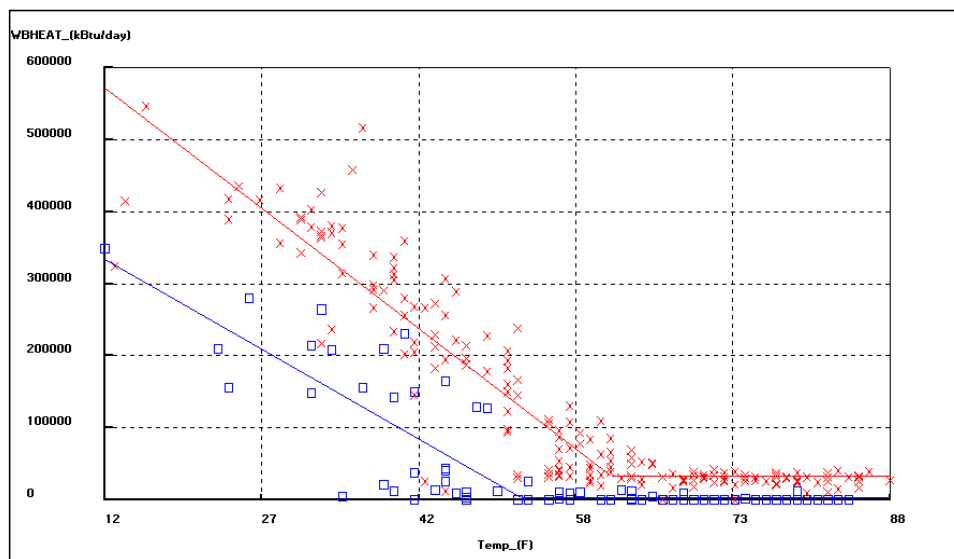


**Figure 6. Example Adjusted Whole-building Analysis (3P Model) for kWh/day ( $R^2 = 0.83$ , CV(RMSE) = 19.5%).**

In another example the hourly steam use (Figure 7) and hourly electricity use (Figure 9) for the U.S.D.O.E. Forrestal Building is modeled with a daily weekday-weekend three parameter, change-point model for the steam use (Figure 8), and an hourly weekday-weekend demand model for the electricity use (Figure 10). To develop the weather-normalized model for the steam use the hourly steam data and hourly weather data were first converted into average daily data, then a three parameter, weekday-weekend model was calculated using the EModel software, which contains similar algorithms as ASHRAE's IMT. The resultant model, which is shown in Figure 8 along with the daily steam, is well described with an  $R^2$  of 0.87, an RMSE of 50,085.95 kBtu/day, and a CV(RMSE) of 37.1%.



**Figure 7. Example Heating Data for Daily Whole-building Analysis (Haberl and Culp, 2004).**



**Figure 8. Example Daily Weekday-weekend Whole-building Analysis (3P Model) for Steam Use (kBtu/day,  $R^2 = 0.87$ , RMSE = 50,085.95, CV(RMSE) = 37.1%). Weekday use (x), weekend use (□)(Haberl and Culp, 2004).**

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